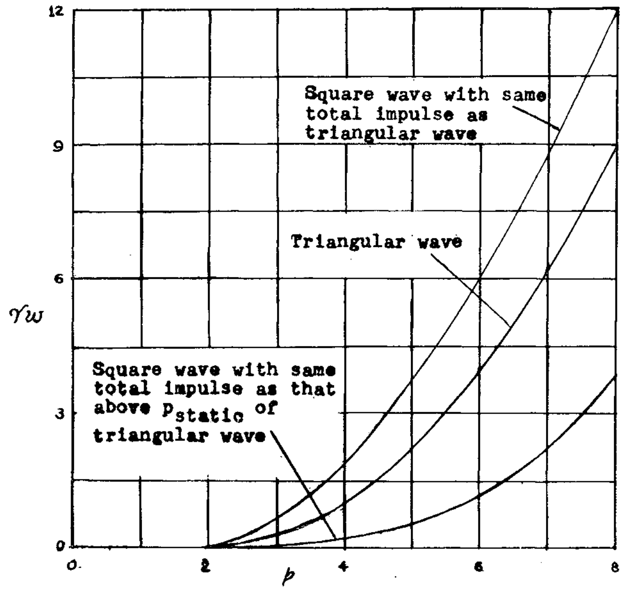

Fig. 3 Comparison of displacements resulting from triangular and square waves.

The condition that $T_1 > 2b$ puts a limitation on the values of p that can be applied, namely,

$$p > 2 + 2^{1/2} \tag{8b}$$

For the squares wave, Fig. 2, a pressure-time relationship is given by

$$p > p_{\text{static}} \qquad 0 < \tau < \tau_1 \tag{9a}$$

$$p = 0 \qquad \tau_1 < \tau < T_2 \tag{9b}$$

where

$$T_2 = p\tau_1/2 \tag{10}$$

Substituting Eqs. (9) into Eq. (5) and integrating subject to the initial and continuity conditions, Eqs. (3) yield

$$\left.\begin{aligned}\gamma\ddot{w} &= p - 2\\ \gamma\dot{w} &= (p-2)\tau\\ \gamma w &= (p-2)(\tau^2/2)\end{aligned}\right\} 0 < \tau < \tau_1 \tag{11a}$$

$$\left.\begin{aligned}\gamma\ddot{w} &= -2\\ \gamma\dot{w} &= (p\tau_1 - 2\tau)\\ \gamma w &= (p\tau_1/2)(2\tau - \tau_1) - \tau^2\end{aligned}\right\} \tau_1 < \tau < T_2 \tag{11b}$$

It should be noted that for the square wave $c = \infty$, and hence $\tau_0 = 0$. The solution for the triangular wave is compared with that of two matching square waves, all with the same peak load. If the square wave has the same total impulse as the triangular wave, we get

$$\tau_1 = b \tag{12a}$$

If the square wave is chosen so that the triangular wave has the same impulse (due to that part of the pressure which exceeds p_{static}) as that of the square wave, we get

$$\tau_1 = (bc - 2)^2/bc^2 \tag{12b}$$

The final displacements due to the triangular wave and the two matching square waves are plotted in Fig. 3. It is seen that, when the square wave has the same total impulse as the triangular wave, the displacement is considerably overestimated. On the other hand, when the waves are chosen so that they have the same impulse due to that part of the pressure which exceeds p_{static}, the displacement is considerably less. Similar results have been obtained by Hodge[3] for the case of circular cylindrical shells.

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Validity of Triple-Point Calculation Applied to Jet Mach Disk

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Recently a general description was presented and published for a method of calculating the size and position of the Mach disk in an underexpanded inviscid jet.[1] The technique is applicable to calculating the mixed subsonic-supersonic flow downstream of the Mach disk. The authors presented the results of a calculation for one particular set of flow conditions.[1] They compared their result with a schlieren picture for the same set of experimental flow conditions. The agreement was good.

To utilize the technique[1] for calculating the position of the Mach disk one must first calculate the inviscid flow field without the presence of the Mach disk (for example, by the method of characteristics). The location of the Mach disk on the interior or jet shock is ascertained by applying the usual triple-point equations and by requiring that the Mach disk be locally normal to the incident flow at the triple point.

Adamson[2] has questioned the range of validity of this approach based upon the results of Kawamura.[3] Kawamura's results indicate that the triple-point calculation does not agree with experiment except for a small range of stagnation-to-ambient-pressure ratios near the value where the Mach disk first occurs. At the highest pressure ratio treated by Kawamura, he was not able to find a triple-point solution consistent with the experimental results. Adamson[2] suggested that the normal triple-point relations cannot be applied to this higher pressure ratio. As a result of this apparent discrepancy, this particular case has been studied in some detail.

The version of the Kawamura paper studied by the present authors did not contain schlieren photos suitable for analysis. The present authors duplicated the experimental apparatus of Kawamura. Results were obtained. The quality of the schlieren photos was not as good as that given by Ladenburg and Bershader[4] for a smaller orifice diameter but with nearly the same pressure ratio as the one that gave Kawamura trouble. The results obtained by the present authors and by Ladenburg and Bershader are similar. It is presumed that they agree also with the results obtained by Kawamura.

Kawamura found the triple-point relations invalid for a total-to-ambient-pressure ratio (p_t/p_s) equal to 5.72. In Fig. 1 is given the shadowgraph result of Ladenburg and Bershader for p_t/p_a equal to 5.72. Also shown in Fig. 1 are the results from the triple-point calculation made by the present authors assuming that the Mach disk is locally perpendicular

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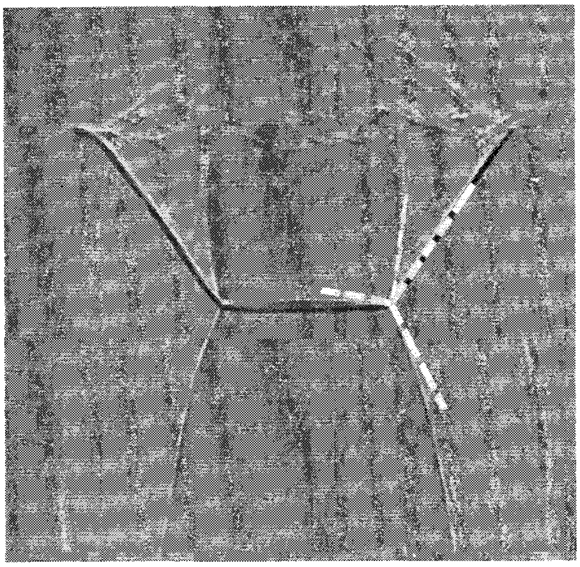

Fig. 1 Shadowgraph of an axially symmetric air jet flowing from a round orifice of 10 mm in diameter at a reservoir gage pressure of 70 psi (reproduced from Ref. 4). Superimposed are calculated slopes for the incident shock, reflected shock, Mach disk, and slip line locally at the triple point.

to the flow at the triple point. The photograph in Fig. 1 shows clearly the curvature of the Mach disk.

The triple-point results presented in Fig. 1 were based on interpolated estimates of the flow-field properties given by Ladenburg, Van Voorhis, and Winckler.[5] The Mach number used is 3.49 (which differs only slightly from the value of 3.51 used by Kawamura; we do not consider this an important difference as the triple-point solution, in general, is not as sensitive to the value of the incident Mach number as it is to the shock angles). The local flow angle used, assuming source flow for this particular example, is 8°. The calculated values for the shock angles appear to be in excellent agreement with the experimental values for the shock angles, in contradiction to the comments of Kawamura.[3]

It is believed that Kawamura failed to find valid triple-point conditions for this particular case because he assumed that the Mach disk was everywhere perpendicular to the axis of symmetry. At small pressure ratios, the Mach disk is small, the curvature is small, and the slope of the Mach disk at the triple point is much more nearly normal to the axis than it is at high-pressure ratios. As a result, Kawamura was able to obtain valid triple-point relations for these less underexpanded conditions by assuming that the shock was everywhere perpendicular to the axis.

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Liquid to Gas Heat Transfer in a Nuclear Reactor

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Among advanced high-temperature reactor concepts, the liquid core reactor has several advantages. However, heat-transfer rates obtainable by bubbling a propellant or working gas through a liquid are not particularly high. The purpose of this note is to point out another mechanism for transferring heat from a liquid to a gas which avoids the difficulties inherent in the bubbling process and might lead to a quite simple reactor design. Such a reactor might be ideal for use in magnetohydrodynamic (MHD) space power and propulsion cycles such as those described in Ref. 1.

Consider a gas flowing over the surface of a hot liquid with a velocity u. Assuming turbulent flow, the heat transfer by convection will be

$$q(\text{convection}) = N_{st}\rho_g u C_p[T(\text{wall}) - T(\text{gas})] \qquad (1)$$

where N_{st} is the Stanton number, ρ_g is the gas density, C_p is the heat capacity, and T is the temperature. Now if the liquid is hot enough to have an appreciable vapor pressure, there will also be a transfer of mass from the liquid to the gas described by a formula similar to that just given for heat transfer. The vapor will carry with it its latent heat L_v, and if the vapor condenses into droplets as a consequence of being cooled by the gas, the net heat transfer from the liquid to the gas by this mechanism will be

$$q\ (\text{vaporization and condensation}) = N_{st} u L_v[\rho_v(\text{wall}) - \rho_v(\text{gas})] \qquad (2)$$

where ρ_v (wall) is the vapor density in equilibrium with the liquid surface, and ρ_v (gas) is the vapor density in the gas. This mechanism of heat transfer to a gas by vaporization and subsequent condensation of a liquid is actually widespread in nature and in industry. The cooling tower is one common example, although the object in this case is to cool the liquid rather than to heat the gas.

In principle very large heat-transfer rates may be obtained in this manner. In particular, if the vapor pressure of the liquid exceeds the ambient pressure and boiling occurs, the rate will no longer be given by Eq. (2) but will certainly exceed it and increase rapidly. Violent boiling will probably lead to increased liquid entrainment in the reactor exhaust, and the limiting boiling rate will be at least partially determined by how much entrainment can be tolerated. In an MHD power or propulsion system, a relatively high rate might be tolerable, whereas in a direct cycle thermal rocket the rate must be very small.

If a very hot fissioning liquid is to be held in a container having a cooled liner there must be compatibility between the materials involved. Also, one must ask whether or not the heat loss involved in cooling the liner is reasonable. It is desirable that the liquid have a low heat conductivity and that the solid liner have a high one. One possible combination is liquid UO_2 in a tungsten liner.

Suppose as an example, that we wish to heat helium at 30 atm pressure to a temperature in the neighborhood of 4000° K. At this pressure it will take approximately 5000°K to make UO_2 boil. The resulting temperature profile through the liquid layer and container of the reactor is shown in Fig. 1. The profile is flat in the region where boiling occurs and falls off to the inner wall temperature, which in this case has been chosen to be 2500°K. The heat-flow equation in the region in which conduction to the wall of the container occurs is $d^2T/dx^2 = Q/\kappa$ where Q is the fission energy release

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